Name ______________________ Section ____________ Date ____________

Laboratory 1

Measurement of Length

PRELABORATORY ASSIGNMENT

Read carefully the entire description of the laboratory and answer the following questions based on the material contained in the reading assignment. Turn in the completed prelaboratory assignment at the beginning of the laboratory period prior to the performance of the laboratory.

1. State the number of significant figures in each of the following numbers:

(a) 37.60 ________

(b) 0.0130 ________

(c) 13000 ________

(d) 1.3400 ________

2. Perform the indicated operations to the correct number of significant figures using the rules for significant figures.

(a) $\begin{array}{r} 37.60 \\ \times\, 1.23 \\ \hline \end{array}$

(b) $6.7\,\overline{)8.975}$

(c) $\begin{array}{r} 3.765 \\ +\ 1.2 \\ +37.21 \\ \hline \end{array}$

3 to 8. Assume the true value of the speed of sound in air at 20°C is 343.5 m/s. Three students named Abe, Barb, and Cal make the following measurements (in m/s) of the speed of sound in air at 20°C:

Abe — 357.4, 339.6, 346.2, 349.2

Barb — 322.6, 324.7, 323.5, 326.9

Cal — 340.6, 347.6, 342.6, 345.8

Calculate the mean, standard deviation from the mean, and standard error for each student's measurements. Comment on the accuracy and precision of each student's measurements, and indicate which students (if any) seem to have a systematic error.

3. Abe Mean = ________ Std Dev from mean = ________ Std Err = ________

4. Barb Mean = ________ Std Dev from mean = ________ Std Err = ________

5. Cal Mean = ________ Std Dev from mean = ________ Std Err = ________

Comment on accuracy, precision, and systematic errors for each student.

6. Abe

7. Barb

8. Cal

9. What class of errors is assumed to be the basis for statistical variations in measurements?

(a) personal (b) systematic (c) random

10. The presence of which class of errors will cause statistical calculations to be completely invalid? (a) personal (b) systematic (c) random

11. Assume σ_{n-1} stands for the standard deviation from the mean. If the assumptions of statistical theory are met, what percentage of measurements is expected to be in the range of the mean $\pm\ \sigma_{n-1}$? ________ % What percentage is expected to be in the range of the mean $\pm\ 2\sigma_{n-1}$? ________ %

Laboratory 1

Measurement of Length

OBJECTIVES

Physics is an experimental science and, as such, it is largely a science of measurement. The measurement of length is of fundamental importance in scientific work; hence, it is fitting to begin experimental work with this type of measurement. In this experiment, repeated measurements of the dimensions of a laboratory table will be used to accomplish the following objectives:

1. Demonstration of the concept of experimental uncertainty in simple measurements using a meter stick
2. Explanation of the specific knowledge gained by repeated measurements of the length and width of a table
3. Application to these measurements of the statistical concepts of mean, standard deviation from the mean, and standard error
4. Demonstration of the propagation of errors by the determination of the uncertainty in the area calculated from the measured length and width
5. Comparison of the error propagation predicted by statistical theory with the error propagation implied by the simpler concept of significant figures

EQUIPMENT LIST

1. 2-m stick
2. Laboratory table

THEORY

In the section entitled "General Laboratory Information" there is a discussion of the subject of systematic errors. It is possible that there might be small systematic errors associated with this experiment. For example, if the meter sticks used in the experiment were made improperly, or have shrunk or expanded since being manufactured, they would be the source of a systematic error in the results. Any such systematic error will be ignored for the purposes of this experiment. Thus, no statements can be made about the accuracy of the experiment.

There will, of course, be random errors in the measurements associated with the fact that it is not possible to interpolate between the smallest marked scale divisions in exactly the same way every time. Therefore, repeated measurements are expected to show some variation about the mean. If there are no personal errors (mistakes) in the process, this variation will be the basis for the determination of the statistical error or uncertainty in the measurement. If systematic errors are present, the calculated mean will not be a good estimate of the true value, but the precision of the results are still given by the standard error. Note, however, that if personal errors are made, the statistical calculations may not be valid.

A central point of experimental work is the idea that it is necessary not only to determine the value of some measured quantity, but also to attempt to determine the uncertainty in that value. In fact, some would claim that a measurement whose uncertainty is completely unknown is of no use whatsoever! The aim of a measurement is not, therefore, to determine the true value of a quantity (because that cannot be done exactly), but rather to set limits within which it is highly probable that the true value lies. The closer these limits can be set, the more reliable is the measurement.

In this experiment it is assumed that the uncertainty in the measurement of the length and width of the table is due to random errors. If this assumption is valid, then the mean of a series of repeated measurements represents the most probable value for the length or width. For a more complete discussion of this idea, refer to the "General Laboratory Information" section.

Consider the general case in which n measurements of the length and width of the table are made. For this experiment 10 measurements will be made, so $n = 10$ for this case, but the equations will be developed for the case in which n can be any chosen value. If L_i and W_i stand for the individual measurements of the length and width, and $\bar{L}$ and $\bar{W}$ stand for the mean of those measurements, the equations relating them are:

$$\bar{L} = (1/n)\sum_{1}^{n} L_i \qquad \bar{W} = (1/n)\sum_{1}^{n} W_i \tag{1}$$

To emphasize what was said above, these values of $\bar{L}$ and $\bar{W}$ represent the most probable value for the true values of the length and width assuming no systematic or personal errors are present.

Information about the precision of the measurement is obtained from the variations of the individual measurements using the statistical concept of the standard deviation as described in the section on "General Laboratory Information." The values of the standard deviation from the mean for the length and width of the table, σ_{n-1}^{L} and σ_{n-1}^{W} are given by the equations:

$$\sigma_{n-1}^{L} = \sqrt{1/(n-1)\sum_{1}^{n}(L_i - \bar{L})^2} \qquad \sigma_{n-1}^{W} = \sqrt{1/(n-1)\sum_{1}^{n}(W_i - \bar{W})^2} \tag{2}$$

If the errors are only random, it should be true that approximately 68.3% of the measurements of length should fall in the range $\bar{L} \pm \sigma_{n-1}^{L}$ and that approximately 68.3% of the measurements of width should fall within the range $\bar{W} \pm \sigma_{n-1}^{W}$.

The precision of the mean for $\bar{L}$ and $\bar{W}$ are given by quantities called the "standard error", α_L and α_W. These quantities are defined by the following equations:

$$\alpha_L = \frac{\sigma_{n-1}^{L}}{\sqrt{n}} \qquad \alpha_W = \frac{\sigma_{n-1}^{W}}{\sqrt{n}} \tag{3}$$

The meaning of α_L and α_W is that if the errors are only random, there is a 68.3% chance that the true value of the length lies within the range $\bar{L} \pm \alpha_L$, and the true value of the width lies within the range $\bar{W} \pm \alpha_W$.

An important problem in experimental physics is the determination of the uncertainty in some quantity that is derived by calculations from other directly measured quantities. For this experiment, consider the area A of the table as calculated from the measured values of the length and width $\overline{L}$ and $\overline{W}$ by the following:

$$A = \overline{L} \times \overline{W} \tag{4}$$

This general problem was discussed in the "General Laboratory Information" section, and the case of the product of two measured quantities is given on page 12 by equation 7. Making the appropriate changes of variables for the present case gives

$$\alpha_A = \sqrt{\overline{L}^2\,\alpha_W{}^2 + \overline{W}^2\,\alpha_L{}^2} \tag{5}$$

This equation gives the standard error of the area of the table, α_A, in terms of the length and width of the table and their associated standard errors. Note that a similar equation applies for standard deviation from the mean:

$$\sigma_{n-1}^{A} = \sqrt{\overline{L}^2\,(\sigma_{n-1}^{W})^2 + \overline{W}^2\,(\sigma_{n-1}^{L})^2} \tag{6}$$

EXPERIMENTAL PROCEDURE

1. Place the 2-m stick along the length of the table near the middle of the width and parallel to one edge of the length. Do not attempt to line up either edge of the table with one end of the meter stick or with any certain mark on the meter stick.
2. Let x stand for the coordinate position in the length direction. Read the scale on the 2-m stick that is aligned with one end of the table and record that measurement in the Data Table as x_1 (meters). Read the scale that is aligned at the other end of the table and record that measurement in the Data Table as x_2 (meters). Note that the smallest-marked-scale division of the stick is 1 mm. *Therefore, each coordinate should be estimated to the nearest 0.1 mm (nearest 0.0001 m).*
3. Do not perform any subtractions to determine values for the length at this time. Calculations of the length will be performed later from the values of x_1 and x_2, but if calculations are performed now to determine the length, it could bias future readings.
4. Repeat steps 1 and 2 nine more times, for a total of 10 measurements of the length of the table. For each measurement, place the 2-m stick on the table with no attempt to align either end of the stick or any particular mark on the stick with either end of the table. To the extent possible, place the stick along the same line of the table each time. The focus of this experiment is to study the limitation in ability to measure some given length, rather than any additional variation caused by the fact that the table most likely does not have exactly the same length at every line across its width. If the goal was to perform the best possible measurement of the table, such variations should be included. However, the interest here is to examine the statistical variation in the measurement of some fixed length; therefore, choose the same line along the table length each time.
5. Perform steps 1 through 4 for 10 measurements of the width of the table. Let the coordinate for the width be given by y and record the 10 values of y_1 and y_2 (meters) in the Data Table. Again place the stick along the same line each time, but make no attempt to align any particular mark on the stick with either edge of the table.

CALCULATIONS

1. After all measurements are completed, perform the subtractions of the coordinate positions to determine the 10 values of the length L_i, and the 10 values of width W_i. Record the 10 values of L_i and W_i in the Calculations Table.

2. Using equations 1, 2, and 3, calculate the mean length $\overline{L}$, the mean width $\overline{W}$, the standard deviations from the mean σ_{n-1}^{L} and σ_{n-1}^{W} and the standard errors α_L and α_W for both the length and width. Initially, calculate $\overline{L}$ and $\overline{W}$ to *seven or eight* significant figures and calculate σ_{n-1}^{L}, σ_{n-1}^{W}, α_L, and α_W to *two* significant figures, but do not record those values in the Calculations Table at this time. Do, however, record all these values somewhere for future use in the calculations. Now record the calculated values of σ_{n-1}^{L}, σ_{n-1}^{W}, α_L, and α_W to *one* significant figure. Next record the values of $\overline{L}$ and $\overline{W}$ so that they have their most significant digit in the same decimal place as the standard error for that quantity. As an example, suppose that 10 measurements of the length have a value of $\overline{L} = 1.352542$ m, a value of $\sigma_{n-1}^{L} = 0.0003$ m, and a value of $\alpha_L = 0.00009$ m. Using the above rules for this example, you would record values of $\alpha_L = 0.00009$, $\sigma_{n-1}^{L} = 0.0003$, and $\overline{L} = 1.35254$. Record your values of $\overline{L}$ and $\overline{W}$ in the Calculations Table to the number of significant figures indicated by the decimal place of the last significant figure in α_L and α_W as described in the example.

3. Using the original values of $\overline{L}$, $\overline{W}$, σ_{n-1}^{L}, σ_{n-1}^{W}, α_L, and α_W, in equations 5 and 6, calculate the standard deviation from the mean, σ_{n-1}^{A} and the standard error, α_A, for the area. Record them in the Calculations Table to *one* significant figure.

4. Using the values of $\overline{L}$ and $\overline{W}$ in equation 4, calculate the area of the table, determining the number of significant figures in the result in two different ways. First, determine the number of significant figures in the area by a procedure similar to the one used above to determine the significant figures in $\overline{L}$ and $\overline{W}$. Let the most significant digit in the calculated area be in the same decimal place as the most significant digit in α_A. Call this value of the area A_1 and record it as A_1 in the Calculations Table.

5. Next, calculate another value for the area, determining the number of significant figures in the result by the following procedure. Use for $\overline{L}$ a value with the number of significant figures contained in each of the values of L_i and use for $\overline{W}$ a value with the number of significant figures contained in each of the values of W_i. Note that this is exactly what the standard rules for significant figures would suggest. Take the product of those values of $\overline{L}$ and $\overline{W}$ using the significant figure rules for multiplication given on page 11 in the "General Laboratory Information" section. Record the result in the Calculations Table as A_2. Note that the only difference between these two values of the area that could result is the number of significant figures, and depending on your precision, there might be no difference at all.

Name ______________________ Section ____________ Date ____________

Laboratory 1

Measurement of Length

LABORATORY REPORT

Data Table

Trial	x_2 (m)	x_2 (m)	y_2 (m)	y_2 (m)
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

Calculations Table

Trial	$L_i = x_2 - x_1$ (m)	$W_i = y_2 - y_1$ (m)
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

$\overline{L}$ = __________ m σ_{n-1}^{L} = __________ m α_L = __________ m

$\overline{W}$ = __________ m σ_{n-1}^{W} = __________ m α_W = __________ m

A_1 = __________ m^2 σ_{n-1}^{A} = __________ m^2 α_A = __________ m^2

A_2 = __________ m^2

SAMPLE CALCULATIONS

QUESTIONS

1. What percentage of your values of L_i fall in the range $\overline{L} \pm \sigma_{n-1}^L$? ______ %
   What percentage fall in the range $\overline{L} \pm 2\sigma_{n-1}^L$? ______ %

2. What percentage of your values of W_i fall in the range $\overline{W} \pm \sigma_{n-1}^W$? ______ %
   What percentage fall in the range $\overline{W} \pm 2\sigma_{n-1}^W$? ______ %

3. According to the theory of random errors, what percentage would be expected for the answers to question 1?

   $\overline{L} \pm \sigma_{n-1}^L$ ______ % $\overline{L} \pm 2\sigma_{n-1}^L$ ______ %

4. Do any of your values of L_i have deviations greater than $3\sigma_{n-1}^L$ from $\overline{L}$? Do any have deviations greater than $3\sigma_{n-1}^W$ from $\overline{W}$? If so, indicate which ones and calculate how many times larger than σ_{n-1}^L or σ_{n-1}^W is the deviation.

5. Based on your answers to questions 1 through 4, are your data reasonably consistent with the assumption that only random errors are present in the experiment? State clearly the basis for your answer.

Using the discussion of standard error in the "General Laboratory Information" section as a guide, state the most probable value of the length, width, and area of the table and their respective uncertainties as determined by statistical theory.

6. length = __________ m ± __________ m

7. width = __________ m ± __________ m

8. area = __________ m^2 ± __________ m^2

The two values for the area, A_1 and A_2, represent two different approaches to determination of the uncertainty in a calculated quantity. The value given as A_1 represents the most correct determination of the uncertainty based on the application of statistical theory to assumed random measurement errors. The value given as A_2 assumes that the uncertainty is in the least significant digit for each individual measurement, but does not explicitly state what the uncertainty is in that digit.

9. For your data, does A_1 have more or fewer significant figures than A_2, or do they have the same number of significant figures?

10. Assuming that A_2 is only an approximation to the more correct A_1, state whether A_2 overestimates or underestimates the uncertainty in the area compared to A_1.