$$v_{iy} = v_i \sin\theta = (72\ m/s)(\sin 60°)$$
$$= (72\ m/s)(0.866) = 62\ m/s$$

How long does it take the cannonball to reach its maximum height?

Solution: When the ball is at its highest point the vertical component of the velocity is zero. At that point the cannonball has stopped moving upward and is just about to begin moving downward. Thus,

$$v_{fy} = v_{iy} + a_y\Delta t$$
$$0\ m/s = 62\ m/s + (-9.8\ m/s^2)(\Delta t)$$
$$\Delta t = \frac{62\ m/s}{9.8\ m/s^2} = 6.3\ s$$

How high does the cannonball rise?

Solution:

$$\Delta s_y = v_{iy}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$
$$= (62\ m/s)(6.3\ s) + \tfrac{1}{2}(-9.8\ m/s^2)(6.3\ s)^2$$
$$= 196\ m$$

What is the cannonball's total time of flight?

Solution: We have already found that the time going up is 6.3 s. To find the time going down:

$$\Delta s_y = v_{iy}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$
$$196\ m = (0\ m/s)(\Delta t) + \tfrac{1}{2}(9.8\ m/s^2)(\Delta t)^2$$
$$\Delta t = \sqrt{\frac{196\ m}{4.9\ m/s^2}} = 6.3\ s$$
$$\text{total time of flight} = 6.3\ s + 6.3\ s = 12.6\ s$$

This answer can also be obtained by multiplying the time required to reach the maximum height by two.

What is the cannonball's range?

Solution:

$$\Delta s_x = v_x\Delta t$$
$$= (36\ m/s)(12.6\ s) = 453\ m$$

QUESTIONS

Base your answers to questions 1 through 4 on the following information: An arrow is shot with an initial velocity of 50. m/s at an angle of 60° from the horizontal.

1. What is the vertical component of the arrow's initial velocity? (1) 25 m/s (2) 43 m/s (3) 50. m/s (4) 58 m/s

2. Neglecting air resistance, after how many seconds does the arrow reach its maximum height? (1) 4.4 s (2) 5.1 s (3) 43 s (4) 420 s

3. How high does the arrow rise? (1) 95 m (2) 190 m (3) 220 m (4) 284 m

4. How far has the arrow traveled horizontally by the time it returns to the ground? (1) 110 m (2) 220 m (3) 380 m (4) 450 m

5. At what angle from the horizontal must a projectile be launched in order to achieve the greatest range? (1) 20° (2) 30° (3) 45° (4) 57.3°

6. At the same moment that a baseball is thrown horizontally by a pitcher, a ring drops vertically off his hand. Which statement about the baseball and the ring is correct, neglecting air resistance? (1) The baseball hits the ground first. (2) The ring hits the ground first. (3) They both hit the ground at the same time.

Uniform Circular Motion.

An object undergoes **uniform circular motion** if it moves along a circular path at a constant speed. Although the magnitude of the object's velocity is constant, the velocity *direction* is constantly changing. A change in the velocity vector of an object means that the object is accelerating. The acceleration experienced by an object in uniform circular motion is called **centripetal acceleration.** Centripetal acceleration $\mathbf{a}_c$ is a vector quantity, directed toward the center of the circle. The magnitude of this acceleration vector is directly proportional to the square of the object's speed and inversely proportional to the radius of its path. Thus,

$$a_c = \frac{v^2}{r}$$

where v is the speed of the object along the circular path and r is the radius of the circle.

Since a circling object is accelerating, there must be a net force acting on it—it is not in equilibrium. The force that causes the centripetal acceleration is called the **centripetal force $\mathbf{F}_c$.** This force acts in the same direction as the centripetal acceleration—toward the center of the circle (Figure 1-39). The magnitude of the centripetal force is obtained from Newton's second law.

$$F_c = ma_c = m\frac{v^2}{r}$$

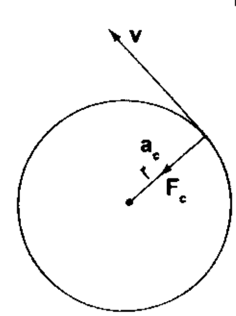


Figure 1-39.

The centripetal force is the net force on the circling object that acts to change the object's direction. The object's speed is constant because there are no tangential forces acting on the object.

Sample Problem

A 1.0-kg ball attached to the end of a rope 0.50 m long is swung in a circle. Its speed along the circular path is 6.0 m/s. Find the centripetal acceleration and force.

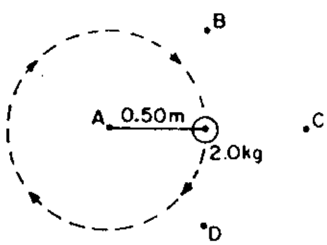
Solution:

$$a_c = \frac{v^2}{r} = \frac{(6.0\ \text{m/s})^2}{(0.50\ \text{m})} = 72\ \text{m/s}^2$$

$$F_c = ma_c = (1.0\ \text{kg})(72\ \text{m/s}^2) = 72\ \text{N}$$

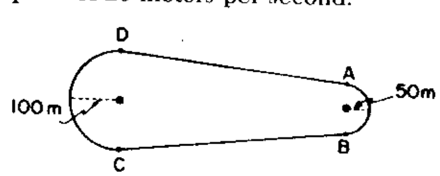
QUESTIONS

Base your answers to questions 1 through 4 on the diagram below, which represents a 2.0-kilogram mass moving in a circular path on the end of a string 0.50 meter long. The mass moves in a horizontal plane at a constant speed of 4.0 meters per second.



1. The force exerted on the mass by the string is (1) 8 N (2) 16 N (3) 32 N (4) 64 N
2. In the position shown in the diagram, the momentum of the mass is directed toward point (1) *A* (2) *B* (3) *C* (4) *D*
3. The centripetal force acting on the mass is directed toward point (1) *A* (2) *B* (3) *C* (4) *D*
4. The speed of the mass is changed to 2.0 meters per second. Compared to the centripetal acceleration of the mass when moving at 4.0 meters per second, its centripetal acceleration when moving at 2.0 meters per second would be (1) half as great (2) twice as great (3) one-fourth as great (4) four times as great

Base your answers to questions 5 through 9 on the diagram below, which represents a flat racetrack as viewed from above, with the radii of its two curves indicated. A car with a mass of 1,000 kilograms moves counterclockwise around the track at a constant speed of 20 meters per second.

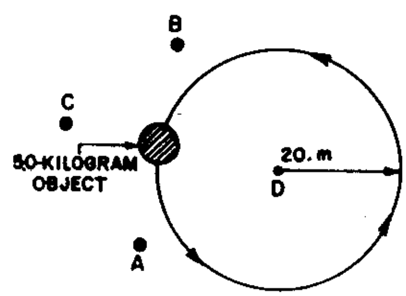


5. The net force acting on the car while it is moving from *A* to *D* is (1) 0 N (2) 400 N (3) 8,000 N (4) 20,000 N
6. The net force acting on the car while it is moving from *D* to *C* is (1) 0 N (2) 200 N (3) 4,000 N (4) 20,000 N
7. If the car moved from *C* to *B* in 20 seconds, the distance *CB* is (1) 100 m (2) 200 m (3) 300 m (4) 400 m
8. Compared to the centripetal acceleration of the car while moving from *B* to *A*, the centripetal acceleration of the car while moving from *D* to *C* is (1) the same (2) twice as great (3) one-half as great (4) 4 times greater

Note that question 9 has only three choices.

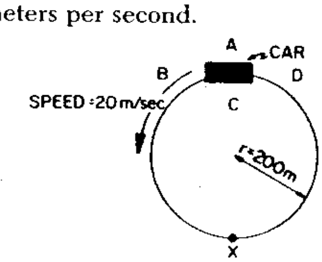
9. Compared to the kinetic energy of the car while moving from *A* to *D*, the kinetic energy of the car while moving from *D* to *C* is (1) less (2) greater (3) the same

Base your answers to questions 10 through 14 on the diagram below, which represents a 5.0-kilogram object revolving around a circular track in a horizontal plane at a constant speed. The radius of the track is 20. meters and the centripetal force on the object is 4.0×10^2 newtons.



10. In the position shown, the object's centripetal acceleration is directed toward point (1) *A* (2) *B* (3) *C* (4) *D*
11. In the position shown, the object's velocity is directed toward point (1) *A* (2) *B* (3) *C* (4) *D*
12. The object's centripetal acceleration is (1) 0.012 m/s^2 (2) 20. m/s^2 (3) 80. m/s^2 (4) 1.0×10^2 m/s^2
13. The object's speed is (1) 20. m/s (2) 40. m/s (3) 60. m/s (4) 90. m/s
14. If the radius of the track is increased, the centripetal force necessary to keep the object revolving at the same speed would (1) decrease (2) increase (3) remain the same

Base your answers to questions 15 through 19 on the diagram below, which represents a car of mass 1,000 kilograms traveling around a horizontal circular track of radius 200 meters at a constant speed of 20 meters per second.



15. When the car is in the position shown, the direction of its centripetal acceleration is toward (1) A (2) B (3) C (4) D

16. The magnitude of the centripetal force acting on the car is closest to (1) 100 N (2) 1,000 N (3) 2,000 N (4) 4,000 N

17. If the speed of the car were doubled, the centripetal acceleration of the car would be (1) the same (2) doubled (3) $\frac{1}{2}$ as great (4) 4 times as great

Note that questions 18 and 19 have only three choices.

18. If additional passengers were riding in the car, at the original speed, the car's centripetal acceleration would be (1) less (2) greater (3) the same

19. As the car moves from its present position to position X, its kinetic energy (1) decreases (2) increases (3) remains the same

KEPLER'S LAWS

Johannes Kepler deduced three laws describing planetary motion. These laws inspired Newton and led to his equations of motion and gravitation.

Kepler's First Law.

Kepler's first law states that the path of each planet is an ellipse with the sun at one focus.

An **ellipse** is defined as a closed curve such that the sum of the distances from any point p on the curve to two fixed points called the foci is constant (Figure 1-40). A circle is an ellipse in which the two foci coincide at the center.

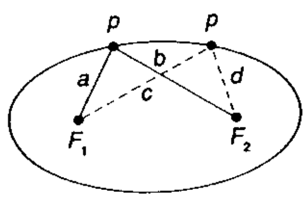


Figure 1-40. F_1 and F_2 are the foci and $a + b = c + d$.

The orbits of some planets are nearly circular while others are distinctly elliptical. For example, Pluto's elliptical orbit will cause it to be closer to the sun than Neptune until 1996. The earth's elliptical orbit brings it closest to the sun in January and furthest away in July. Many comets have elongated elliptical paths with the sun at one focus and the other focus far beyond the solar system (Figure 1-41).

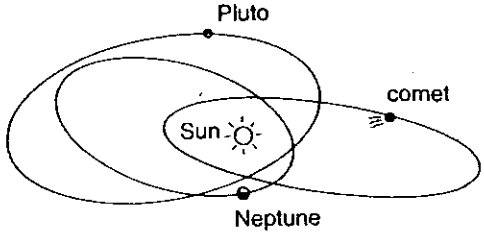


Figure 1-41.

Kepler's Second Law.

Kepler's second law states that each planet moves in such a way that an imaginary line drawn from the sun to the planet sweeps out equal areas in equal periods of time. For example, the wedge-shaped sectors A_1, A_2, and A_3 in the ellipse illustrated in Figure 1-42 are all equal in area. If an imaginary line connecting the planet to the sun passes through area A_1 in one week, it passes through areas A_2 and A_3 in one week each as well. This means that the planet moves faster when it is closer to the sun. As a planet gets closer to the sun, its gravitational potential energy decreases and its kinetic energy increases.

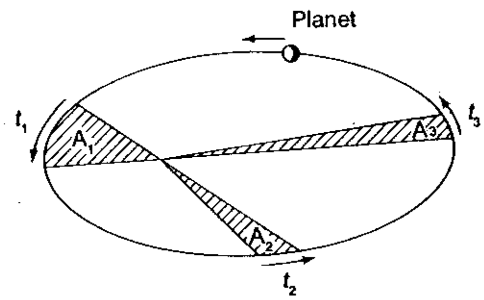


Figure 1-42. The planet moves faster when it is closer to the sun; $t_1 = t_2 = t_3$ and $A_1 = A_2 = A_3$.

Kepler's Third Law.

Kepler's third law states that the ratio of the mean radius of orbit cubed (R^3) to the orbital period squared (T^2) is a constant for all the planets. This relationship is expressed mathematically as

$$\frac{R^3}{T^2} = k$$

where k is a constant. Kepler's third law is applicable to any group of satellites that orbit around a given body. The value of k depends on the mass of the particular body being orbited. In the case of the planets orbiting the sun, k is equal to $3.35 \times 10^{18}\ m^3/s^2$. As the mass of the object being orbited decreases, k decreases. For example, the value of k for objects orbiting the earth is $1.02 \times 10^{13}\ m^3/s^2$.

SATELLITE MOTION

A **satellite** is defined as any body revolving around a larger body. The nine planets are satellites of the sun, and the sixteen moons of Jupiter are satellites of Jupiter. The earth's satellites include the moon and the artificial objects placed in orbit about the earth.

Newton was the first to compare the motion of a satellite around the earth to the motion of a projectile. He concluded that a satellite is simply a projectile that "falls freely" toward the earth. We know that the greater the horizontal component of a projectile's velocity, the greater the horizontal distance it will travel before hitting the ground. It follows that if a projectile has a great enough horizontal velocity, the curved path of its motion will match the curvature of the earth and it will rotate around, instead of fall into, the earth. If a satellite is too close to the earth, the drag of the atmosphere will slow it down and it will spiral inward toward the earth. If, on the other hand, its speed is too great, the satellite will spiral outward and escape from the earth's gravitational pull. The minimum speed an object must have to escape the influence of a body's gravitational pull is called the **escape velocity**.

Apparent Weightlessness.

Astronauts in orbiting spaceships experience a state of "weightlessness" even though the earth's gravity still pulls them toward the earth. Indeed, it is this pull that provides the centripetal force that maintains the orbit. Why then does an astronaut's weight not register on a scale, and why do all the objects in the spaceship float freely? These effects occur because the spaceship is falling toward the earth together with the astronaut, the scale, and all the objects aboard. If the astronaut released a glove, for example, it would not fall to the floor because the floor, the glove, and the astronaut are all falling at the same rate.

Geosynchronous Orbit.

Communication and weather satellites are usually placed in **geosynchronous orbits**. A geosynchronous orbit is one in which the satellite's orbital period is equal to the period of the earth's rotation about its own axis (24 hours). A satellite in geosynchronous orbit remains over the same spot on the earth's equator.

Kepler's third law can be used to determine the radius of orbit needed for an artificial satellite to have a desired orbital period. Since the value of R^3/T^2 is the same for all earth satellites (both natural and artificial), we can simply equate R^3/T^2 for the moon and R^3/T^2 for the satellite. For example,

$$\frac{R^3_{\text{shuttle}}}{T^2_{\text{shuttle}}} = \frac{R^3_{\text{moon}}}{T^2_{\text{moon}}} = k_{\text{earth}} = 1.02 \times 10^{13}\ \text{m}^3/\text{s}^2$$

We conclude that in order to be in geosynchronous orbit, a spaceship must orbit at a distance from the earth's center equal to approximately 6.6 times the earth's radius.

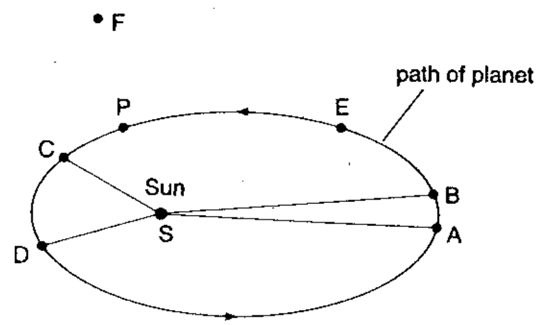
QUESTIONS

1. The path of a planet around the sun is best described as (1) a circle (2) an ellipse (3) a parabola (4) an epicycle

2. With respect to a planet's orbit, the sun is situated at (1) the center (2) one of the foci (3) a nodal point (4) a geocentric point

3. Until 1996 Neptune is farther from the sun than Pluto because Pluto (1) has switched orbits with Neptune temporarily (2) orbits Neptune (3) is denser than Neptune (4) has a very elongated orbit

Questions 4–10 are based on the following diagram of a planet orbiting the sun. It takes the planet one month to travel from point A to point B, and one month to travel from point C to point D.



4. Which of the following statements is correct? (1) arc AB = arc CD (2) length SC = length SA (3) area SCD = area SAB (4) potential energy at C = potential energy at A

5. At which point does the planet have the greatest kinetic energy and speed? (1) A (2) C (3) E (4) the speed is the same at all of the above

6. If this planet is the earth, when is it located at point C? (1) January (2) June (3) July (4) September

7. If the planet's mass were suddenly doubled, the period of its revolution in orbit would (1) decrease (2) increase (3) remain the same

8. When the planet is at point P, the direction of the planet's velocity is toward point (1) S (2) F (3) C (4) B

9. The direction of the planet's acceleration at point P is toward point (1) S (2) E (3) F (4) B

10. If the mass of the sun were suddenly to increase, then the value of R^3/T^2 would (1) decrease (2) increase (3) remain the same

11. What is true of a satellite in geosynchronous orbit? (1) It remains in the same position over a point on the equator. (2) It remains in a fixed position between the earth and the moon. (3) It remains in a fixed position between the earth and the sun. (4) Its period of revolution does not equal the earth's period of rotation.

12. The radius of orbit for an artificial satellite around the earth may be determined by equating R^3/T^2 for the satellite with R^3/T^2 for the (1) earth (2) moon (3) sun (4) other planets

13. What occurs if an orbiting satellite's speed exceeds escape velocity? (1) It spirals back toward earth. (2) It achieves a geosynchronous orbit. (3) It spirals outward away from the earth. (4) Any of the above are possible.